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Nonlinear interaction and reflection of waves in vapor–liquid bubbly mixtures

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Abstract

A mathematical model was developed for investigation of waves in bubbly liquids with planar, cylindrical and spherical symmetry. The problems of reflection of waves from free surface and rigid wall and interaction of waves were considered. It is shown that the reflection of pressure wave from free surface in bubbly mixture is considerably different than in the case of pure liquid.

For the reflection of shock waves from rigid wall, the nonlinear enhancement of wave was established. It is shown that in the system under high static pressure, nonlinearity during the reflection from rigid wall manifest itself stronger. It was established that decreasing of void fraction and increasing of initial bubble radius lead to the increasing of the value of maximum pressure at the wall surface.

Keywords: Vapor–liquid mixture; Bubbles; Wave reflection

1. Introduction

Wave propagation in bubbly liquids was considered in a number of works reviewed in [1,2]. It can also be mentioned [3–6], where the structure of steady shock waves and evolution of nonstationary waves were studied.

In the present work, the system of equations was generalized for cylindrical and spherical one-dimensional case. Calculations for simplicity were made for planar waves only. The reflection of waves from free surface and rigid wall was considered. The interaction of waves was also investigated.

1.1. Basic assumptions

The system of equations describing wave processes in bubbly liquids. Wave processes in a vapor–liquid bubbly mixtures are considered here using continuum mechanics methods under the following basic assumptions:

(i) the distances over which the flow parameters (for example, oscillatory wavelengths) vary significantly are much larger than the distances between the bubbles, which are themselves much larger than the bubble diameters (i.e. the volume fraction of the vapor phase is small enough, $\alpha_v \leqslant 0.1$);
(ii) the mixture is locally monodispersed, i.e. in each material volume all the bubbles are spherical and are of the same radius;
(iii) viscosity and thermal conduction are important only in the processes of interphase interaction and, in particular, in bubble pulsations;
(iv) nucleation, fragmentation, interaction and coagulation of the bubbles are absent;
(v) the velocities of the macroscopic motion of the phases coincide. The last assumption allows us to describe bubble volume changes, temperature distributions around the bubbles, condensation and evaporation in terms of the spherically symmetrical model using the equations for bubble radial pulsations and radial thermal conduction of the liquid. This assumption originates from the fact that for vapor bubbles the

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Nomenclature

R	bubble radius
α	volume concentration of phase
υ	longitudinal velocity
w	radial velocity
p	pressure
ρ	density
j	rate of phase transition per unit interfacial surface
t	time
n	number of bubbles per unit volume
σ	coefficient of surface tension
x	Eulerian longitudinal coordinate
ξ	Lagrangian coordinate
B	gas constant
T	temperature
l	specific heat of evaporation
c	sound speed
λ	thermal conductivity
γ	specific heat ratio
C	specific heat
q	heat flux
φ	correction coefficient
a	thermal diffusivity
τ	$\frac{c_* t}{R_0}$
r_*	$\frac{r}{R}$
ξ_*	$\frac{\xi\sqrt{\alpha_{e0}\alpha_{vo}}}{R_0}$
x_*	$\frac{x\sqrt{\alpha_{e0}\alpha_{vo}}}{R_0}$
P	$\frac{p}{p_0}$
θ	$\frac{T}{T_0}$
R_*	$\frac{R}{R_0}$
J	$\frac{j}{\rho_e^0 c_*}$
c_*	$\sqrt{\frac{p_0}{\rho_e^0}}$
W	$\frac{w_{e\sigma}}{c_*}$
V	$\frac{\upsilon\sqrt{\alpha_{e0}\alpha_{vo}}}{C_*}$
ϕ	$\frac{\rho}{\rho_e^0}$
M	$\frac{\rho_v^0}{\rho_e^0}$
Pe	$\frac{c_* R_0}{a_e}$, Peclet number
Δp_e	$\frac{p_e - p_0}{p_0}$
S	$\frac{2\sigma}{R_0 p_0}$
C_ρ	$\frac{\rho_{vo}^0}{\rho_e^0}$
C_B	$\frac{B_v T_0}{l}$
C_L	$\frac{C_l T_0}{l}$
K	$\frac{\Delta p}{\Delta p_0 + \Delta p_e}$

Subscripts

l	liquid
v	vapor
0	initial equilibrium state
s	at saturation state
σ	on bubble surface
∞	conditions at infinity

role of the interphase heat and mass transfer becomes greater than for gas bubbles, and the two velocity effects are therefore less significant in the background of thermal dissipation [1,2].

Under the assumption listed above, the vapor–liquid medium can be considered within the frameworks of a model of two interacting and interpenetrating continuous media, viz., the carrier liquid and the vapor phase [1].

In the Lagrange system of coordinates (ξ, t), the equations of changes of phase's masses, the equations for a change in the mass of an individual bubble and conservation of momentum of the mixture for one-dimensional motions with planar, cylindrical or spherical symmetry (correspondingly $m = 1, 2, 3$) are as follows:

$$\frac{\partial \rho_l}{\partial t} + \frac{\rho \rho_l}{\rho_0}\left(\frac{x}{\xi}\right)^{m-1}\frac{\partial \upsilon}{\partial \xi} + \frac{\rho_l(m-1)}{x}\upsilon = -4\pi R^2 nj,$$
$$\frac{\partial \rho_v}{\partial t} + \frac{\rho \rho_v}{\rho_0}\left(\frac{x}{\xi}\right)^{m-1}\frac{\partial \upsilon}{\partial \xi} + \frac{\rho_v(m-1)}{x}\upsilon = 4\pi R^2 nj,$$
$$\frac{\partial}{\partial t}\left(\frac{4}{3}\pi R^3 \rho_v^0\right) = 4\pi R^2 j,$$
$$\frac{\partial \upsilon}{\partial t} + \frac{1}{\rho_0}\left(\frac{x}{\xi}\right)^{m-1}\frac{\partial p}{\partial \xi} = 0,$$
$$\frac{\partial x}{\partial t} = \upsilon,$$
$$\rho = \rho_l + \rho_v, \quad p = p_l\alpha_l + (p_v - 2\sigma/R)\alpha_v \approx p_l,$$
$$\alpha_l + \alpha_v = 1, \quad \rho_i = \alpha_i \rho_i^0, \ (i = \mathrm{v}, \mathrm{l}),$$
$$\alpha_v = \frac{4}{3}\pi R^3 n, \qquad (1.1)$$

where the subscripts $i = \mathrm{l}, \mathrm{v}$ refer to the parameters of the liquid and vapor, respectively; the subscript 0 refers to the parameters of the initial equilibrium state; $\alpha_i, p_i, \rho_i, \rho_i^0$ are the volume fraction, pressure, mean and true densities of the ith phase, respectively; R is the bubble radius; j is the rate of phase transition per unit interfacial surface ($j > 0$ for evaporation and $j < 0$ for condensation); υ is the longitudinal velocity; n is the number of bubbles per unit volume; σ is the coefficient of surface tension; ξ is the Lagrangian; x is the Eulerian longitudinal coordinates, respectively, and t is the time.

Note that by virtue of assumption (i) ($\alpha_v \ll 1$) from (1.1), it follows that the average pressure in the mixture practically coincides with the pressure in the liquid phase ($p \approx p_e$).

The system of hydrodynamic Eq. (1.1) will be closed if the equation of state, the condition of the simultaneous deformation of the phases and the equation for determining the phase transition rate j are assigned.

The propagation, interaction and reflection of pressure waves of moderate intensities can be considered under the following additional assumptions:

(a) the carrier liquid phase is incompressible:

$$\rho_e^0 = \text{const.}, \tag{1.2}$$

(b) the vapor obeys the equation of state of a perfect gas, and being in the saturation state it obeys the Clapeyron–Clausius equation:

$$p_v = \rho_v^0 B T_v; \quad \frac{dT_v}{dp_v} = \frac{T_v}{l\rho_v^0}\left(1 - \frac{\rho_v^0}{\rho_l^0}\right) \tag{1.3}$$

(c) the bubble is uniform.

Here T is the absolute temperature, B is the gas constant, l is the specific heat of evaporation.

The assumption that the carrier liquid is incompressible is valid when the wave velocity v, relative to the medium before the front, and the volume fraction of the vapor phase satisfy the conditions $(v/c_l)^2 \ll 1$ and $\alpha_v \gg \alpha_* = p_0/\rho_l^0 c_l^2$, respectively [1], where c_l is the sound speed in the liquid.

Under normal conditions, $p \sim 0.1$ MPa, the above conditions hold for most liquids when the volume fraction of the vapor phase $\alpha_v \geqslant 10^{-2}$ (since $\alpha_* \leqslant 10^{-4}$).

Transfer processes in bubbly liquids are determined by the distributions of microparameters near inhomogeneities [1]. A possible model is one that employs the concept of a cell with a test bubble in it at any Lagrangian point. The cell dimension is determined by the volume fractions of the phases and equals to $R\alpha_v^{-1/3}$, the cell centre coinciding with the centre of the test bubble. The distributions of microparameters inside a cell are described by the equations for the corresponding microprocesses with the boundary conditions on the test bubble surface (which determine the interphase interaction) and on the external boundary of the cell [7]. Consider a spherically symmetric test bubble with its centre at a point x, the microparameters around the bubble being dependent of time t, the position of the bubble centre x, and the distance r of a microparticle from the centre.

To determine the temperature and heat-flux distributions, we use the equation of heat conduction around the bubble. The phase transition rate j may be found from the boundary conditions on the bubble surface. In the absence of a macroscopic heat flux in the carrying phase, that condition on the cell boundary should reflect the cell adiabaticity.

The system of equations describing the distribution of the microparameters around the test bubble and the boundary conditions have the form

$$\begin{aligned}
&C_l\rho_l^0\left(\frac{\partial T_l}{\partial t} + w_{l\sigma}\frac{R^2}{r^2}\frac{\partial T_l}{\partial r}\right) = \frac{1}{r^2}\frac{\partial}{\partial r}\left(\lambda_l r^2\frac{\partial T_l}{\partial r}\right),\\
&r = R(t): \quad T_l = T_v, \ \ jl = -q_v - q_l,\\
&\qquad\qquad q_l = -\lambda_l\frac{\partial T_l}{\partial r},\\
&r_{\text{cell}} = R(t)\alpha_v^{-1/3}: \quad \frac{\partial T_l}{\partial r} = 0,
\end{aligned} \tag{1.4}$$

where w is the velocity of radial motion, C_l is the specific heat of the liquid, λ is thermal conductivity, q_e and q_v are the heat fluxes to the liquid and vapor, respectively, from the interface. The subscript σ refers to the parameters at the interface.

Within the framework of a uniform bubble containing a saturated vapor, the heat flux $q_v = \lambda_v \frac{\partial T_v}{\partial r}\big|_R$ spent on a change in vapor saturation temperature caused by a pressure change is nonzero because the uniform-bubble model corresponds to the asymptotic condition $\lambda_v \to \infty$, $\partial T_v/\partial r \to 0$. In this case $\lambda_v \partial T_v/\partial r \neq 0$, because we have indeterminacy of the type $\infty \cdot 0$.

For q_v to be calculated in terms of the model of a uniform bubble filled with saturated vapor, we use the equation of the heat flowing to the vapor phase. Substituting the total derivative of the saturated-vapor temperature by the derivative of the pressure according to (1.3), and integrating this equation with respect to r within the limits from 0 to R, we arrive at [8].

$$q_v = \frac{1}{3}R\left[\frac{C_{pv}}{l}\left(1 - \frac{\rho_v^0}{\rho_e^0}\right) - 1\right]\frac{\partial p_v}{\partial t} = \frac{C_s T_v}{3l}R\frac{\partial p_v}{\partial t}, \tag{1.5}$$

where C_s is the vapor specific heat along the phase equilibrium curve [9]

$$C_s = C_p - T\left(\frac{dp}{dT}\right)_s \frac{\partial}{\partial T}\left(\frac{1}{\rho}\right)_p. \tag{1.6}$$

For most liquids, particularly for water, under normal conditions ($p \sim 0.1$ MPa) $C_s < 0$. This means that for vapor to remain in a saturated state when it is compressed, heat should be abstracted from it. For water $C_s = 0$ at $p \approx 3$ MPa.

The pressures of the phases and the bubble radius are related by the condition of simultaneous deformation, as described by the Rayleigh–Plesset equation [1,10]:

$$\begin{aligned}
&(1-\varphi_1)R\frac{\partial w_{e\sigma}}{\partial t} + \frac{3}{2}(1-\varphi_2)w_{l\sigma}^2\\
&\quad = \frac{p_v - p_l - 2\sigma/R}{\rho_l^0} - \frac{2jw_{l\sigma}}{\rho_l^0},\\
&\frac{dR}{dt} = w_{l\sigma} + j/\rho_l^0,\\
&\varphi_1 = 1.5\frac{\alpha_v^{1/3} - \alpha_v}{\alpha_l}, \quad \varphi_2 = \frac{\alpha_v^{1/3}(2+\alpha_v) - 3\alpha_v}{\alpha_l}.
\end{aligned} \tag{1.7}$$

In this equation, correction coefficients φ_1 and φ_2, are introduced taking into account "nonsingleness" of the bubble. The phase transition terms also were taken into account. These corrections φ_1 and φ_2 characterize the difference of the fictitious pressure p_∞ at infinity from the average pressure p_e in the liquid. By adding boundary and initial conditions, the system of Eqs. (1.1)–(1.7) will be closed. As initial condition we took the condition of equilibrium of vapor–liquid system at initial hydrostatic pressure p_0.

2. Transformation of the original system of equations to the form suitable for numerical integration

Let us transform (1.1) to a form suitable for numerical integration for the case of a one-dimensional wave with planar, cylindrical or spherical symmetry.

From continuity equations for liquid and vapor phase (1.1), we can obtain

$$\frac{\partial\alpha_v}{\partial t}=\frac{\alpha_l\rho}{\rho_0}\left(\frac{x}{\xi}\right)^{m-1}\frac{\partial\upsilon}{\partial\xi}+\frac{m-1}{x}\alpha_l\upsilon+\frac{4\pi R^2nj}{\rho_l^0},$$
$$\frac{\partial\alpha_v}{\partial t}=-\frac{\alpha_v\rho}{\rho_0}\left(\frac{x}{\xi}\right)^{m-1}\frac{\partial\upsilon}{\partial\xi}-\frac{m-1}{x}\alpha_v\upsilon+\frac{4\pi R^2nj}{\rho_v^0}-\frac{\alpha_v}{\rho_v^0}\frac{\partial\rho_v^0}{\partial t}. \tag{2.1}$$

By equating the right sides of the equations, we obtain

$$\frac{\rho}{\rho^0}\left(\frac{x}{\xi}\right)^{m-1}\frac{\partial\upsilon}{\partial\xi}=4\pi R^2nj\left(\frac{1}{\rho_v^0}-\frac{1}{\rho_l^0}\right)-\frac{m-1}{x}\upsilon-\frac{\alpha_v}{\rho_v^0}\frac{\partial\rho_v^0}{\partial t}. \tag{2.2}$$

From the equation of conservation of mass of an individual bubble (1.1), we can obtain expression for the derivative of real density of the vapor

$$\frac{\partial\rho_v^0}{\partial t}=-\frac{3\rho_v^0}{R}\left(\frac{dR}{dt}-\frac{j}{\rho_v^0}\right). \tag{2.3}$$

If we substitute (2.3) to (2.2), we obtain

$$\frac{\partial\upsilon}{\partial\xi}=\frac{\rho_0}{\rho}\left(\frac{\xi}{x}\right)^{m-1}\frac{3\alpha_v}{R}w_{e\sigma}-\frac{m-1}{x}\left(\frac{\xi}{x}\right)^{m-1}\frac{\rho_0\upsilon}{\rho}. \tag{2.4}$$

By adding continuity equations for vapor and liquid phases (1.1), we obtain

$$\frac{\partial\rho}{\partial t}+\frac{\rho^2}{\rho_0}\left(\frac{x}{\xi}\right)^{m-1}\frac{\partial\upsilon}{\partial\xi}+\frac{\rho(m-1)\upsilon}{x}=0. \tag{2.5}$$

If we substitute (2.4) to (2.5), we obtain

$$\frac{\partial\rho}{\partial t}=-\frac{3\alpha_v\rho w_{e\sigma}}{R}. \tag{2.6}$$

By differentiating the equation of conservation of momentum with respect to ξ, Eq. (2.4) with respect to t and equating the mixed derivatives, $\partial^2\upsilon/\partial t\partial\xi$ and $\partial^2\upsilon/\partial\xi\partial t$, we obtain, if (1.7) is taken into account, the following differential equation for the average pressure:

$$\frac{\partial^2p}{\partial\xi^2}-D\frac{\partial p}{\partial\xi}=-N,$$
$$D=\frac{1}{\rho^0}\frac{\partial\rho_0}{\partial\xi}+\frac{m-1}{\xi}\left[1-2\frac{\rho_0}{\rho}\left(\frac{\xi}{x}\right)^m\right],$$
$$N=\frac{\rho_0^2}{\rho}\left(\frac{\xi}{x}\right)^{2(m-1)}\left\{\frac{3\alpha_v}{R^2(1-\varphi_1)}\left[\frac{p_v-p_l-2\sigma/R}{\alpha_l\rho_l^0}+2(1-\varphi_1)w_{l\sigma}j/\rho_l^0+(1-4\varphi_1+3\varphi_2)\frac{w_{l\sigma}^2}{2}\right]+(m-1)\frac{\upsilon}{x}\left[m\frac{\upsilon}{x}-\frac{6\alpha_vw_{l\sigma}}{R}\right]\right\}. \tag{2.7}$$

Note that this equation does not contain derivatives with respect to t. This significantly simplifies the numerical integration.

Eq. (2.7) represents an elliptic equation. Note that, according to this equation, pressure disturbances propagate with an infinite velocity. This is the consequence of the incompressibility of the carrier liquid which transmits pressure disturbances. The influence of the bubbles and vapor properties is exhibited through the function $M=M(R_0,w_{l\sigma},p_v,p)$, in which R_0, $w_{l\sigma}$ and p_v can be determined from (1.7) and the following equation of mass for an individual bubble:

$$\frac{\partial p_v}{\partial t}=-\frac{3\gamma p_v}{R\gamma_*}\left(\frac{q_e}{l\rho_v^0}+\frac{\partial R}{\partial t}\right),$$
$$\gamma_*=1+(\gamma-1)\left(1-\frac{C_{pv}T_s}{l}\right)\left[1-\frac{C_{pv}T_s}{l}\left(1-\frac{\rho_v^0}{\rho_l^0}\right)\right], \tag{2.8}$$

where γ is the specific heat ratio. The infinite velocity of disturbance propagation in the carrier liquid is the frozen sound speed in a given two-phase dispersion medium.

Eqs. (2.4), (2.6) and (2.7) allow us to determine the velocity and pressure fields of the mixture at fixed instants through the known fields of the remaining parameters.

If we combine Eqs. (2.4)–(2.8) with equations of microproblem (1.3)–(1.7) then we will obtain the complete system of equations for study of nonsteady one-dimensional mixtures (with planar, cylindrical or spherical symmetry).

In particular case, when $m=1$ (one-dimensional planar wave), these equations will coincide with [4].

In the case of $m\neq1$, these equations contain Eulerian coordinate $x(t,\xi)$.

In this case, the relationship between the Eulerian and Lagrangian coordinates is expressed by the following formula:

$$\frac{\partial}{\partial t}x(t,\xi)=\upsilon. \tag{2.9}$$

Let us use dimensionless parameters

$$\tau=\frac{C_*t}{R_0},\quad r_*=\frac{r}{R},\quad \xi_*=\frac{\xi\sqrt{\alpha_{l0}\alpha_{vo}}}{R_0},$$
$$x_*=\frac{x\sqrt{\alpha_{l0}\alpha_{vo}}}{R_0},\quad P_i=\frac{p_i}{p_0},\quad \theta_i=\frac{T_i}{T_0},$$

$$R_* = \frac{R}{R_0}, \quad J = \frac{j}{\rho_1^0 C_*}, \quad C_* = \sqrt{\frac{p_0}{\rho_1^0}},$$
$$W = \frac{w_{e\sigma}}{C_*}, \quad V = \frac{\upsilon\sqrt{\alpha_{e0}\alpha_{vo}}}{C_*},$$
$$\phi = \frac{\rho}{\rho_e^0}, \quad M = \frac{\rho_v^0}{\rho_e^0}. \tag{2.10}$$

The dimensionless equations contain the following independent nondimensional parameters:

$$Pe = \frac{C_* R_0}{a_1}, \quad \alpha_{vo}, \quad \gamma, \quad \Delta p_e = (p_e - p_0)/p_0,$$
$$S = \frac{2\sigma}{R_0 p_0}, \quad C_\rho = \frac{\rho_{vo}^0}{\rho_1^0}, \quad C_B = \frac{B_v T_0}{l}, \quad C_L = \frac{C_1 T_0}{l}. \tag{2.11}$$

These parameters characterize the influence on the process liquid thermal conductivity (Pe), void fraction (α_{vo}), specific heat ratio of the vapor (γ), wave intensity (Δp_e), capillary effects (S), small relative density of vapor (C_ρ), heat capacities of phases and latent heat of vaporization (C_B, C_L).

For convenience we will omit subscript * for nondimensional parameters.

The problem was solved by a combination of modified Euler method with the sweep method. For simplification only planar waves was considered ($m = 1$).

3. Results and discussion

To investigate the interaction and reflection of plane nonsteady shock waves and pulse disturbances in vapor–liquid bubbly media, we used the closed system of Eqs. (1.3)–(1.7), (2.4)–(2.8). The corresponding mathematical problems consisted in finding solutions of this system, subject to the following initial and boundary conditions at specified cross-sections for the volume of mixture chosen ($\xi = 0$, $\xi = \mathrm{L}$):

$$\left.\begin{aligned} &p_1 = p_0, \quad p_v = p_0 + \tfrac{2\sigma}{R_0}, \quad R = R_0, \\ &\upsilon = w_{1\sigma} = 0, \quad T_e = T_v = T_0, \end{aligned}\right\} \quad t = 0,$$
$$P = f_B(t) \text{ or } \left.\frac{\partial p}{\partial \xi}\right|_{\xi=0} = \phi_B(t) \quad (\xi = 0), \tag{3.1}$$
$$P = f_L(t) \text{ or } \left.\frac{\partial p}{\partial \xi}\right|_{\xi=\mathrm{L}} = \phi_L(t) \quad (\xi = \mathrm{L}).$$

The propagation of short pressure delta pulses was modeled by assuming different laws of a rapid pressure change which correspond to a linear rise and a linear drop of the pressure in the zone of decreasing pressure. For a delta pulse, the function $f(t)$ is of the form

$$f(t) = \begin{cases} p_0 + (p_e - p_0)t/t_1, & t < t_1, \\ [t(p_0 - p_e) + p_e t_2 - p_0 t_1]/(t_2 - t_1), & t_1 \leqslant t \leqslant t_2, \\ p_0, & t_2 < t, \end{cases} \tag{3.2}$$

where $\Delta p_e = p_e - p_0$ is the maximum value of the pressure jump in the pulse or its amplitude, t_1 and t_2 are determined by the duration of the initial pulse.

Let us consider the initial pressure $p_0 = 0.1$ MPa. For the reflection of shock waves from free surface, the boundary condition was

$$p(L, t) = p_0. \tag{3.3}$$

Figs. 1 and 2 present the results of calculation of the pulses reflections from the free surface. The values of initial bubble radius, void fraction α_{vo}, and characteristics of pulses were varied. The pressure disturbances disappear after some time after reflection. Intensity of the pulse and characteristics of bubbly mixture have an effect on velocity and dispersion of pressure disturbances reflection. In particular, initial void fraction has significant effect on dispersion. This is because the velocity of wave propagation

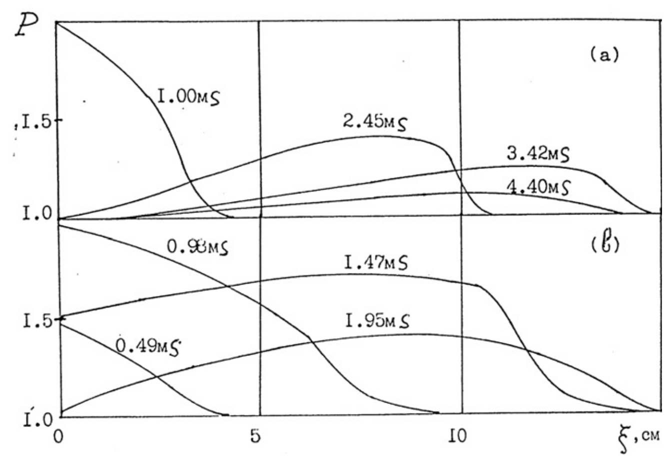

Fig. 1. The reflection of the pulse from the free surface in the vapor–liquid bubbly mixture ($t_1 = 1$ ms, $t_2 = 2$ ms, $R_0 = 1$ mm, $p_e = 2p_0$): (a) $\alpha_{vo} = 0.05$ and (b) $\alpha_{vo} = 0.01$.

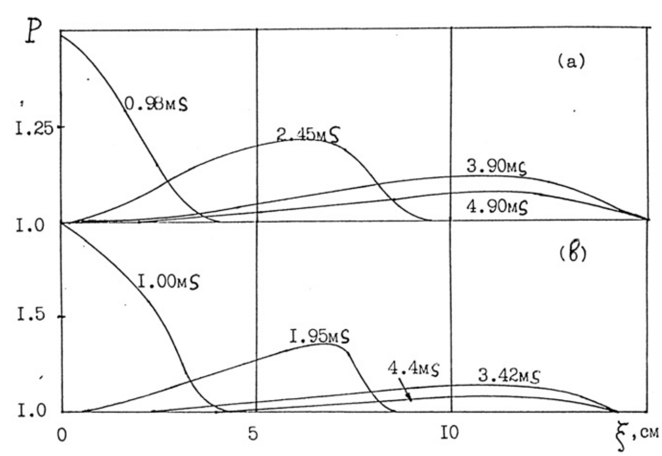

Fig. 2. The reflection of the pulse from the free surface in the vapor–liquid bubbly mixture ($R_0 = 1$ mm, $\alpha_{vo} = 0.05$): (a) $t_1 = 1$ ms, $t_2 = 2$ ms, $p_e = 1.5p_0$ and (b) $t_1 = 1$ ms, $t_2 = 1.25$ ms, $p_e = 2p_0$.

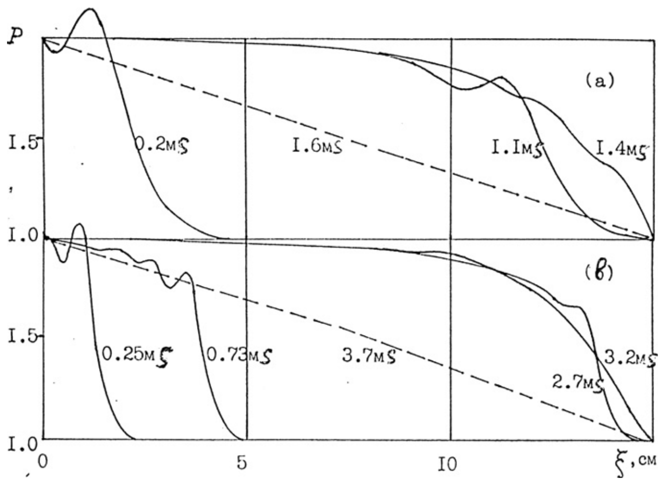

Fig. 3. The reflection of the shock waves from the free surface in the vapor–liquid bubbly mixture: (a) $\alpha_{vo} = 0.01$, $R_0 = 1$ mm and (b) $\alpha_{vo} = 0.05$, $R_0 = 1$ mm.

increases when void fraction decrease. For the case presented in Fig. 1b, the duration of the pulse is greater than the time of traveling of the pulse until free surface. But the character of reflection does not change dramatically.

The effect of R_0 on reflection is similar. The reflection of shock waves is presented in Fig. 3. After some time the pressure profile along the mixture approaches the linear dependence. So, introduction of the bubbles into a slightly compressible liquid changes the character of reflection of waves from free surface dramatically. Bubbly liquid, unlike pure liquid, is much more sensible to the strain stress. The reflection of pressure waves from free surface in the case of "pure" liquid ($\alpha_{vo} < 10^{-8}$) was discussed in [11]. In the case of "pure" liquid, negative pressure can occur after reflection of waves. In the case considered here when $\alpha_{vo} \sim 10^{-3}$–10^{-2}, the pressure can be a little less than the initial value p_0 during very short time intervals only (after reflection).

For the case presented in Fig. 3a, the minimum value of the mixture pressure after wave reflection is about $0.98p_0$. This value decreases when α_{vo} decreases.

Let us consider the reflection of waves from a rigid wall. In this case, the boundary condition has the form

$$\frac{\partial p}{\partial \xi}(L,t) = 0. \tag{3.4}$$

The typical situation characterizing the reflection of shock waves from rigid wall in liquid with vapor bubbles is presented in Fig. 4. At the nonstationary stage, the pressure profile has a characteristic splash.

The value of Δp_e was varied. So, the different intensity waves were initiated at both boundaries. The exceeding of parameter Δp_{ξ^*} of each wave of some critical value, leads to a nonlinear intensification during the passing of the waves through each other. Here the current amplitude

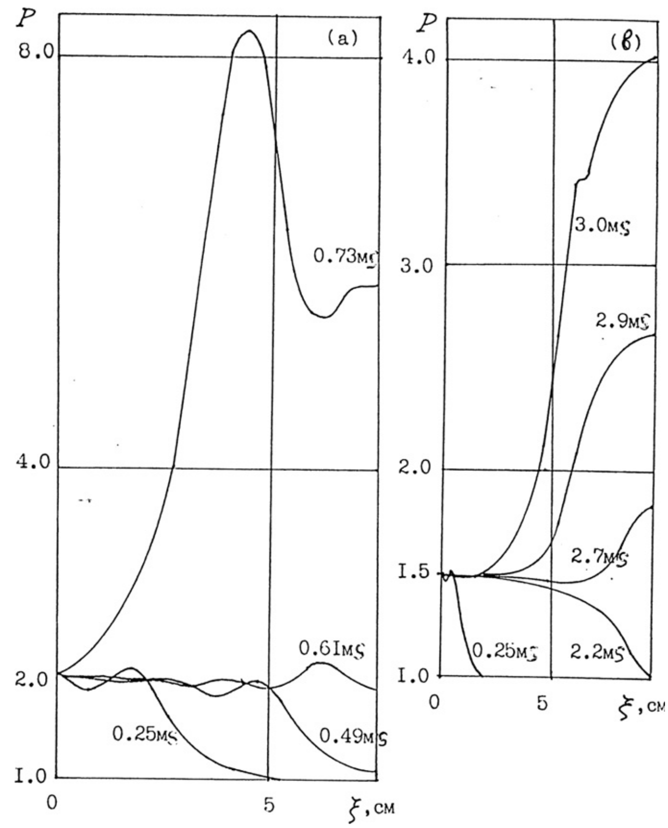

Fig. 4. The reflection of the wave from the rigid wall in the vapor–liquid bubbly mixture: (a) $\alpha_{vo} = 0.01$, $R_0 = 1$ mm and (b) $\alpha_{vo} = 0.05$, $R_0 = 1$ mm.

$\Delta p_{\xi^*} = \max(p - p_0)$. ξ_* equals to 0 or 1 in dependence of the place of wave initiation. Let us introduce the parameter

$$K = \frac{\Delta p}{\Delta p_0 + \Delta p_1}, \tag{3.5}$$

where Δp is the maximum value of the difference between current and initial pressure of mixture during the interaction. In the case of simple superposition of wave amplitudes $K = 1$. In the case of nonlinear intensification of waves $K > 1$.

Calculations show that an increase of α_{vo} leads to a decrease of K. The small velocity of disturbances propagation corresponds to the large values of α_{vo}. For this reason, the increasing of α_{vo}, which means the increasing of vapor content in the mixture, leads to greater dissipation of the wave energy.

So, the reflection of pulses from rigid wall can be non-acoustical and considerably weaker of linear reflection. This is related to the phase transitions creating the big energy dissipation of pulses.

The possibility of the anomalous increasing of pressure in the reflected wave was verified in a number of experimental works. An example of such intensification is given in [12] where during the reflection of shock wave of initial intensity $\Delta p_e = 0.04$ MPa the pressure greater than 0.4 MPa was fixed in the reflected wave. Approximately the same intensification was obtained in calculations on the base of the present model.

For investigation of the effect of high value of the initial static pressure on the reflection of waves, calculations were made for the following conditions:

$p_0 = 1.0$ MPa, $\alpha_{vo} = 0.02$, $R_0 = 1$ mm.

Fig. 5 represents the propagation and reflection of pulse from rigid wall ($p_e = 3p_0$, $t_0 = 0.015$ ms, $t_* = 0.15$ ms) and calculated "oscillogramme" of the pressure at the wall. One can see that the wave is reaching the wall for the time around 0.1 ms. The amplitude of the pressure oscillations at the wall reaches $16p_0$.

The pulsation of the pressure, caused by the incident wave, corresponds to the bubbles radial oscillations near the rigid wall.

The interaction of two single waves in boiling water with vapor bubbles under high static pressure was considered ($p_0 = 1.0$ MPa).

The intensities at the left and right boundaries were equal $p_{max}/p_0 = 3$. Duration of pulses was $t_* = 0.15$ ms. The initial distance between the pulses was 0.2 m. After some time, the disturbances initiated at the left and right boundaries propagated in the form of single waves, moving to meet each other.

Fig. 6 represents the formed at the 0.15 ms pulses. During the motion to meet each other, dissipation caused by heat and mass transfer leads to the decrease of pulses amplitude.

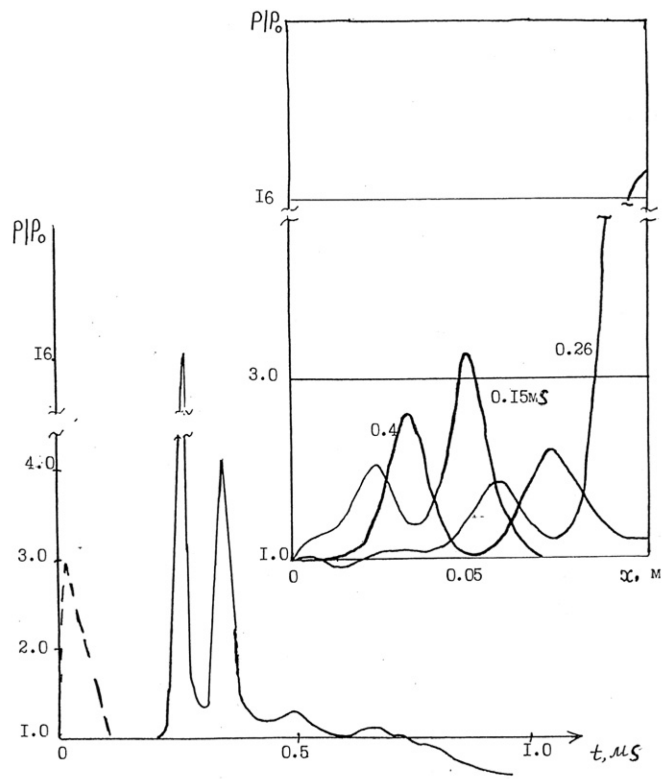

Fig. 5. The reflection of the pulse from the rigid wall in the vapor–liquid bubbly mixture and the calculated "oscillogramme" of the pressure at the wall: $p_0 = 1.0$ MPa, $\alpha_{vo} = 0.02$, $R_0 = 1$ mm, $p_{max} = 3p_0$, $t_* = 0.15$ ms.

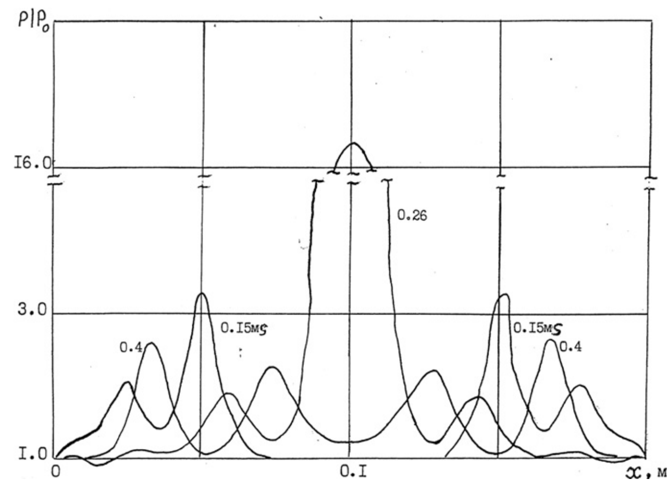

Fig. 6. The interaction of strong pulses in the vapor–liquid bubbly mixture: $p_0 = 1.0$ MPa, $\alpha_{vo} = 0.02$, $R_0 = 1$ mm, $p_{max} = 3p_0$, $t_* = 0.15$ ms.

At the moment $t = 0.26$ ms, the pulses meet and intensity at that time reaches the value $16p_0$. In case the interaction is according to the linear theory, the amplitude of the formed pulses would be $p_{max} = 5p_0$. So, nonlinear effects lead to more than three times higher enhancement of the amplitude of the total pulse.

Acoustical properties of bubbly and pure liquid are totally different. The pure liquid can be studied in the frameworks of linear theory for the waves of intensity up to hundreds of atmospheres. In bubbly liquids, nonlinear effects can manifest themselves even in weak waves with the pressure intensity $\frac{p_e}{p_0} \sim 0.1$. After reflection the wave propagates along the mixture which was already compressed. For this reason void fraction was decreased, and compressibility of the mixture is also decreased.

The singularities of wave propagation in bubble screen were discussed in [13].

4. Conclusion

The reflection of waves from free surface and rigid wall in vapor–liquid bubbly mixtures were considered. The interaction of waves in such media was also studied. It is shown that the reflection of wave from free surface in bubbly mixture is considerably different than in the case of pure liquid. For the reflection of waves from rigid wall, the nonlinear enhancement of wave was established.

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